Certification of Completion-Based Infeasibility Proofs

Teppei Saito   René Thiemann

February 22, 2024

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1. unraveling for infeasibility (using #908)
2. ground completeness/joinability for infeasibility (using #853)

Unraveling for Infeasibility

Definition (infeasibility problem)

specification of infeasibility problem is as follows

input: conditional (oriented) TRS $\mathcal{R}$ and queries $s_1 \to^* t_1, \ldots, s_n \to^* t_n$

output: whether there is a substitution σ s.t. $s_1\sigma \to_{\mathcal{R}}^* t_1\sigma, \ldots, s_n\sigma \to_{\mathcal{R}}^* t_n\sigma$

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3. solve word problem by completion (e.g., Knuth–Bendix completion)

Step 1: Turn Query into Ground One

Proposition

for fresh symbols T and F, the following statements are equivalent

- $s_1 \to^* t_1, \ldots, s_n \to^* t_n$ *is infeasibile under CTRS* $\mathcal{R}$
- $T \to^* F$ *is infeasible under* $\mathcal{R} \cup \{T \to F \Longleftarrow s_1 \to^* t_1, \ldots, s_n \to^* t_n\}$

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- grounding query is essential for final step (refutation by completion)

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remarks

- grounding query is essential for final step (refutation by completion)
- one can virtually apply rule-inlining by Sternagels (CADE 2017) to query

Example: Apply Step 1 to COPS#908 (ARI#763)

query $x < \mathsf{min}(y) \to^* \mathsf{true},\ x < \mathsf{min}(y) \to^* \mathsf{false}$ under CTRS $\mathcal{R}$

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\begin{aligned}
x < 0 &\to \mathsf{false} \\
0 < \mathsf{s}(y) &\to \mathsf{true} \\
\mathsf{s}(x) < \mathsf{s}(y) &\to x < y \\
\mathsf{min}(\mathsf{cons}(x, \mathsf{nil})) &\to x \\
\mathsf{min}(\mathsf{cons}(x, xs)) &\to x && \Longleftarrow && x < \mathsf{min}(xs) \to^* \mathsf{true} \\
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is transformed into new query $\mathsf{T} \to^* \mathsf{F}$ under CTRS

$$
\mathcal{R} \cup \{\mathsf{T} \to \mathsf{F} \Longleftarrow x < \mathsf{min}(y) \to^* \mathsf{true},\ x < \mathsf{min}(y) \to^* \mathsf{false}\}
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Step 2: Unraveling

Definition (Marchiori 2005; Claessen and Smallbone 2018; Ōi 2020)

unraveling $\mathbb{U}$ maps each rule $\ell \to r \Longleftarrow s_1 \to^* t_1, \ldots, s_n \to^* t_n$ into either

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- $\{\ell \to r\}$, or
- $\{\ell \to U(s_1, \ldots, s_n, \vec{x}),\ U(t_1, \ldots, t_n, \vec{x}) \to r\}$

where U is function symbol and $\vec{x}$ are variables

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$\to_{\mathcal{R}} \subseteq \to^+_{\mathbb{U}(\mathcal{R})}$ *for all CTRSs* $\mathcal{R}$ *and unravelings* $\mathbb{U}$, *where* $\mathbb{U}(\mathcal{R}) = \bigcup_{\rho \in \mathcal{R}} \mathbb{U}(\rho)$

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we may choose same U for different rules, or ignore conditions

Example: Unraveling COPS#908 (Failure)

part of conditional rules:

$$\mathsf{min}(\mathsf{cons}(x, xs)) \to x \quad \Longleftarrow \quad x < \mathsf{min}(xs) \to^* \mathsf{true}$$
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use U_1 and $[x, xs]$ for the first, and U_2 and $[x, xs]$ for the second

$$\mathsf{min}(\mathsf{cons}(x, xs)) \to \mathsf{U}_1(x < \mathsf{min}(xs), x, xs) \qquad \mathsf{U}_1(\mathsf{true}, x, xs) \to x$$
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but Moca fails in the next step (or fails to find complete presentation)

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then Moca succeeds on transformed problem afterwards

Step 3: Disproof by Completion

at this point we have ground query $s \to^* t$ and TRS $\mathcal{R}$

Proposition

$s \to^* t$ is infeasible if there is confluent $\mathcal{S}$ with $\mathcal{R} \subseteq \leftrightarrow^*_{\mathcal{S}}$ and s, t not $\mathcal{S}$-joinable

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$s\sigma = s \to^*_\mathcal{R} t = t\sigma \quad \Longrightarrow \quad s \leftrightarrow^*_\mathcal{S} t \quad \Longrightarrow \quad$ s and t are joinable $\lightning$

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- we can use any completion (Knuth–Bendix/maximal/with termination tool)

Demo and Summary of Part 1

demo!

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Ground-Completeness/Joinability

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observation: goal is ground in the last step (for solving word problem)

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let $\mathcal{E}$ be equations and $>$ reduction order

Definition (ordered rewrite system or OTRS)

$$\mathcal{E}_> = \{\ell\sigma \to r\sigma \mid \ell \approx r \in \mathcal{E} \cup \mathcal{E}^{-1}, \ell\sigma > r\sigma\}$$

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remark: extended critical pair lemma is nice characterization of ground-confluence

Moca completely ignores condition parts of COPS#853 (ARI#709)

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x < 0 &\to \mathsf{false} \\
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\mathsf{app}(x : xs, ys) &\to x : \mathsf{app}(xs, ys) \\
\mathsf{split}(x, \mathsf{nil}) &\to \mathsf{nil} \\
\mathsf{split}(x, y : ys) &\to \mathsf{pair}(xs, y : zs) & \Longleftarrow \mathsf{split}(x, ys) \to^* \mathsf{pair}(xs, zs), \ldots \\
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\mathsf{qs}(\mathsf{nil}) &\to \mathsf{nil} \\
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but successfully finds ground-confluent OTRS $\mathcal{E}_{>}$ (!?)

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- required properties of closure operation are described in axiomatic way

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todo: LPO closure for COPS#{809, 853, 857, 930} (in March)

Comments by Audience

- NN: Ōi's refinement was already observed by Gmeiner, Nishida and Gramlich (WST 2013)
- NN: ConCon's tree automata approach also completely ignores conditions
- NH: simulating TCAP by completion-based approach?
- RT: partial unraveling (leaving some conditions) could be useful?